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NORMALIZATION OF BISTATIC RADAR RETURN

by

Donald E. Barrick

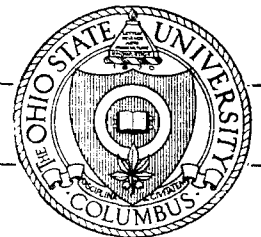
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Grant Number	NsG-213-61
Investigation of	Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-Like Surfaces
Subject of Report	Normalization of Bistatic Radar Return
Submitted by	Donald E. Barrick Antenna Laboratory Department of Electrical Engineering
Date	15 January 1964

ABSTRACT

2 4 / 1 3

Theoretically, the scattering cross-section per unit area of an arbitrary surface can be determined by measurement of the power received, as long as an integration of the incident power density over the surface illuminated can be performed. The class of surfaces considered in this report have an average height which is planar, and the antennas are highly directional and equidistantly placed at various incidence and scattering angles above the surface. The integration is performed for various values of incidence and scattering using the digital computer, and tables of this normalized integral are presented. These tables may then be used to determine the approximate bistatic scattering cross section for such a surface from measurements of the scattered power.

Author

CONTENTS

	Page
I. GENERAL PROBLEM FORMULATION	1
II. SOLUTION OF INTEGRAL FOR A PLANE SURFACE	4
III. VALUES OF THE INTEGRAL $I(\theta_1, \theta_2, \phi_2)$	8
IV. DETERMINATION OF SCATTERING CROSS SECTION FROM THE TABLES	13

NORMALIZATION OF BISTATIC RADAR RETURN

I. GENERAL PROBLEM FORMULATION

One useful parameter in the scattering of waves by extended surfaces is the scattering cross section. This scattering cross section, or, more conveniently, the scattering cross section per unit area of surface, $\sigma_0(i, s)$, is a function of the angles of incidence (i) and scattering (s) from any particular element of surface area. Therefore, whenever the transmitting and receiving antennas are relatively near the surface and since the angles of incidence and scattering will differ for the different elements of area making up the surface, this scattering cross section, $\sigma_0(i, s)$, is a function of the position of the surface.

In this report, the basis for normalizing bistatic surface return measurements made with pencil beam antennas is presented. A set of tables is provided to interpret a measured power ratio (power returned from surface/power returned from a standard target with a known scattering cross section) as a radar cross section per unit area.

Consider a surface which may or may not contain some degree of roughness. In Fig. 1 the mean height of the surface is considered to be constant, i. e., a plane through which the x-z plane of the coordinate system is shown.* The origin of the system is chosen equidistant from both antennas at a distance L . The antennas are aligned with their axes pointing towards the origin. The x-y plane is chosen coincident with the plane containing the incident wave and the normal to the surface.

*Note that this does not restrict the definition of scattering cross section to be applicable only to the case of a mean planar surface. However, for the sake of clarity, and since the planar surface is the case to be treated later in the report, development of the scattering cross section will be done with reference to this figure.

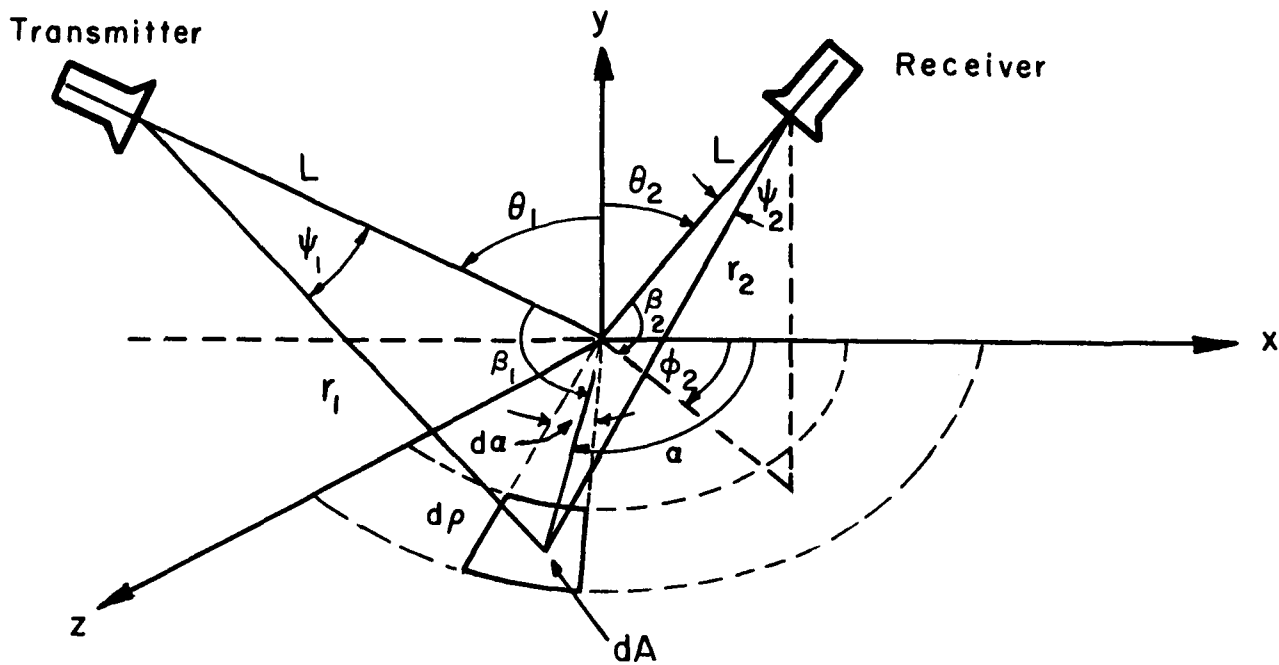


Fig. 1. Geometry of scattering experiment.

The power density of the incident wave at a distance r_1 from the transmitting antenna is given by the expression

$$S_i = \frac{P_T G_{MT} f_T(\psi_1)}{4\pi r_1^2}.$$

P_T is the total power emitted by the transmitter, $f_T(\psi_1)$ is the normalized "power gain function" of the antenna,¹ and G_{MT} is the maximum gain of the antenna. Note that the antenna considered here has a gain function rotationally symmetrical about the antenna axis.*

* This is often not exactly the case with actual antennas, but an average may be taken at various rotated positions for each conical angle ψ , so that an average $f(\psi)$ may be computed for the antenna.

The scattering cross section per unit area, σ_o , may be defined by the following relation, where dS_s is the power density of the scattered wave at a distance r_2 from the scattering surface element dA :

$$\sigma_o(i, s) = \frac{4\pi r_2^2 dS_s}{dA S_i} .$$

Therefore,

$$dS_s = \frac{\sigma_o(i, s) S_i dA}{4\pi r_2^2} = \frac{P_T G_{MT} f_T(\psi_1) \sigma_o(i, s) dA}{(4\pi)^2 r_1^2 r_2^2} .$$

The relationship between the scattered power density, dS_s , at the receiver and the increment of power received by the antenna, dP_R , due to scattering by the given increment of surface area dA is given by $dP_R = A_R dS_s$, where A_R is the aperture or receiving cross section of the antenna. Since

$$A_R = \frac{\lambda^2 G_{MR} f_R(\psi_2)}{4\pi} ,$$

where $f_R(\psi_2)$ and G_{MR} have the same significance for the receiving antenna as the corresponding quantities had for the transmitting antenna, the equation giving the increment of power received, scattered from the increment of surface area dA , is

$$dP_R = P_T \frac{\lambda^2}{(4\pi)^3} G_{MR} G_{MT} \frac{\sigma_o(i, s) f_T(\psi_1) f_R(\psi_2) dA}{r_1^2 r_2^2} .$$

The total power received, scattered by the entire surface, is therefore

$$P_R = P_T \frac{\lambda^2}{(4\pi)^3} G_{MR} G_{MT} \int_{\substack{\text{SURF.} \\ \text{ILLUMINATED}}} \frac{f_T(\psi_1) f_R(\psi_2) \sigma_o(i, s)}{r_1^2 r_2^2} dA .$$

Since it is based on the definition of $\sigma_0(i, s)$, this expression is valid for a surface having any arbitrary shape. It is, in fact, an integral equation relating received power and surface scattering properties. In principle, by measuring the transmitted and received power for all antenna positions, one can solve this integral equation and determine the scattering cross section, $\sigma_0(i, s)$. Practically, however, solution of this equation is impractical, if not impossible, unless certain restrictions and assumptions are permitted. In the following section, the case mentioned previously of the mean planar surface will be treated, and the restrictions and assumptions applicable will be developed.

II. SOLUTION OF INTEGRAL FOR A PLANE SURFACE

In certain scattering problems, the effective area illuminated can have an average surface height which is planar, as shown in Fig. 1. Polar coordinates ρ and α are used to describe the mean planar surface used as a reference. The first approximation made is that if the surface is rough, the root-mean square height of a point on the surface above the reference plane is much less than the distance from the origin to the antenna. In this case, integration can be carried out over the plane reference surface instead of the actual rough surface, since ρ and α will not be appreciably different for a point on the reference surface and the corresponding point above or below it on the actual surface, in the illuminated area.

Since the antennas are highly directional, $f_T(\psi_1)$ and $f_R(\psi_2)$ fall off rapidly, and, in most cases, have their half-power points only one or two degrees on either side of the antenna axis. Thus, only a relatively small area around the coordinate origin is illuminated, and, in this area, the angles of incidence do not change appreciably between any two increments of area. Therefore, the second approximation made is that the scattering cross section, $\sigma_0(i, s)$, is effectively constant over the illuminated area and may be removed from under the integral sign. In this case, $\sigma_0(i, s) \approx \sigma_0(\theta_1, \theta_2, \phi_2)$,

$$\therefore P_R = P_T \frac{\lambda^2}{(4\pi)^3} G_{MR} G_{MT} \sigma_0(\theta_1, \theta_2, \phi_2) \int_0^{2\pi} \int_0^\infty \frac{f_T(\psi_1) f_R(\psi_2)}{r_1^2 r_2^2} \rho \, d\rho \, d\alpha$$

At this point r_1 , r_2 , $f_T(\psi_1)$ and $f_R(\psi_2)$ will be expressed in terms of ρ and α . The steps will be listed briefly and the reasoning will follow.

From the figure,

$$\left. \begin{aligned} \cos \beta_1 &= -\sin \theta_1 \cos \alpha \\ \cos \beta_2 &= \sin \theta_2 \cos \phi_2 \cos \alpha \\ &\quad + \sin \theta_2 \sin \phi_2 \sin \alpha \\ &= \sin \theta_2 \cos (\phi_2 - \alpha) \end{aligned} \right\} \begin{array}{l} \text{Definition of cosine of} \\ \text{angle by dot product of} \\ \text{two vectors} \end{array}$$

$$\left. \begin{aligned} \therefore r_1^2 &= L^2 + \rho^2 - 2L\rho \cos \beta_1 \\ &= L^2 + \rho^2 + 2L\rho \sin \theta_1 \cos \alpha \\ \therefore r_2^2 &= L^2 + \rho^2 - 2L\rho \cos \beta_2 \\ &= L^2 + \rho^2 - 2L\rho \sin \theta_2 \cos (\phi_2 - \alpha) \end{aligned} \right\} \begin{array}{l} \text{Law of Cosines} \end{array}$$

The antenna gain patterns may be determined by experiment and normalized to give plots of $f_T(\psi_1)$ and $f_R(\psi_2)$ versus ψ_1 and ψ_2 . Then approximate mathematical curves may be fitted to these points. A fairly satisfactory function in the case of many pencil beam antennas is

$$\begin{aligned} f_T(\psi_1) &= e^{-A(1-\cos \psi_1)} \approx e^{-A \frac{\psi_1^2}{2}} ; \\ f_R(\psi_2) &= e^{-B(1-\cos \psi_2)} \approx e^{-B \frac{\psi_2^2}{2}} , \end{aligned}$$

where A and B are large positive constants. With the antennas considered here, it was found that $A = B \approx 1100$. However, A and B can vary from this considerably without appreciably affecting the value of the integral. Since $f_T(\psi_1)$ and $f_R(\psi_2)$ are significant only at small angles (they fall off 3 db at $\psi = 2^\circ$ in this case), in this range, $\psi_1 \approx \sin \psi_1$ and $\psi_2 \approx \sin \psi_2$.

$$\left. \begin{aligned} \sin \psi_1 &= \frac{\rho}{r_1} \sin \beta_1 \\ \sin \psi_2 &= \frac{\rho}{r_2} \sin \beta_2 \end{aligned} \right\} \text{ Law of Sines } ,$$

$$\begin{aligned} \therefore \psi_1 &\approx \sin \psi_1 = \frac{\rho}{r_1} \sin [\cos^{-1} (-\sin \theta_1 \cos \alpha)] \\ &= \frac{\rho}{r_1} \sqrt{1 - \sin^2 \theta_1 \cos^2 \alpha} \end{aligned}$$

$$\psi_1^2 \approx \frac{\rho^2 (1 - \sin^2 \theta_1 \cos^2 \alpha)}{L^2 + \rho^2 + 2L\rho \sin \theta_1 \cos \alpha}$$

$$\begin{aligned} \psi_2 &\approx \sin \psi_2 = \frac{\rho}{r_2} \sin [\cos^{-1} (\sin \theta_2 \cos(\phi_2 - \alpha))] \\ &= \frac{\rho}{r_2} \sqrt{1 - \sin^2 \theta_2 \cos^2 (\phi_2 - \alpha)} \end{aligned}$$

$$\psi_2^2 \approx \frac{\rho^2 (1 - \sin^2 \theta_2 \cos^2 (\phi_2 - \alpha))}{L^2 + \rho^2 - 2L\rho \sin \theta_2 \cos (\phi_2 - \alpha)} ,$$

$$\therefore f_T(\psi_1) \approx e^{-\frac{A}{2} \frac{\rho^2 (1 - \sin^2 \theta_1 \cos^2 \alpha)}{L^2 + \rho^2 + 2L\rho \sin \theta_1 \cos \alpha}} .$$

$$f_R(\psi_2) \approx e^{-\frac{B}{2} \frac{\rho^2 (1 - \sin^2 \theta_2 \cos^2 (\phi_2 - \alpha))}{L^2 + \rho^2 - 2L\rho \sin \theta_2 \cos (\phi_2 - \alpha)}} ,$$

$$\begin{aligned}
& \therefore \int_0^{2\pi} \int_0^{\infty} \frac{f_T(\psi_1) f_R(\psi_2)}{r_1^2 r_2^2} \rho \, d\rho \, d\alpha \\
&= \int_0^{2\pi} \int_0^{\infty} \frac{\rho \, e^{-\frac{A}{2} \frac{\rho^2 (1 - \sin^2 \theta_1 \cos^2 \alpha)}{L^2 + \rho^2 + 2L\rho \sin \theta_1 \cos \alpha} - \frac{B}{2} \frac{\rho^2 (1 - \sin^2 \theta_2 \cos^2 (\phi_2 - \alpha))}{L^2 + \rho^2 - 2L\rho \sin \theta_2 \cos (\phi_2 - \alpha)}}{[L^2 + \rho^2 + 2L\rho \sin \theta_1 \cos \alpha] [L^2 + \rho^2 - 2L\rho \sin \theta_2 \cos (\phi_2 - \alpha)]} \, d\rho \, d\alpha .
\end{aligned}$$

All lengths in the integral can be normalized by a change of variable to a dimensionless ratio, $x = \rho / L$,

$$\begin{aligned}
& \therefore \int_0^{2\pi} \int_0^{\infty} \rho \frac{f_T(\psi_1) f_R(\psi_2)}{r_1^2 r_2^2} \, d\rho \, d\alpha \\
&= \frac{1}{L^2} \int_0^{2\pi} \int_0^{\infty} x \, e^{-\frac{A}{2} \frac{x^2 (1 - \sin^2 \theta_1 \cos^2 \alpha)}{1 + x^2 + 2x \sin \theta_1 \cos \alpha} - \frac{B}{2} \frac{x^2 (1 - \sin^2 \theta_2 \cos^2 (\phi_2 - \alpha))}{1 + x^2 - 2x \sin \theta_2 \cos (\phi_2 - \alpha)}} \frac{dx \, d\alpha}{[1 + x^2 + 2x \sin \theta_1 \cos \alpha] [1 + x^2 - 2x \sin \theta_2 \cos (\phi_2 - \alpha)]} \\
&= \frac{1}{L^2} I(\theta_1, \theta_2, \phi_2) .
\end{aligned}$$

At this point, values of the integral $I(\theta_1, \theta_2, \phi_2)$ were computed at various scattering angles θ_1 , θ_2 , and ϕ_2 using the 7090 Digital Computer and SCRATRAN programming. The upper limit on the integral in x was set at 0.15 instead of infinity, since, at this value, and with angles θ_1 and θ_2 less than 60° , ψ_1 and ψ_2 are always greater than 5° , making the magnitude of the integrand negligible. At angles θ_1 and θ_2 of 80° , a slight error, exceeding no more than 8%, is present due to this integral limit on x .

The values of the integral $I(\theta_1, \theta_2, \phi_2)$ are tabulated in the next section and their use is discussed in Section IV.

III. VALUES OF THE INTEGRAL $I(\theta_1, \theta_2, \phi_2)$

EVALUATION OF SURFACE INTEGRAL APPEARING IN RECEIVER/TRANSMITTER GAIN FOR VARIOUS ANTENNA POSITIONS

THETA 1= 0 DEG.		HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.	0 DEG.	30 DEG.0
0 DEG.	0.0030216	0.0030462	0.0031196	0.0032430	0.0034163	0.0036352	0.0038852	0.0041335	0.0043251		
20 DEG.	0.0030216	0.0030458	0.0031183	0.0032398	0.0034098	0.0036235	0.0038658	0.0041042	0.0042865		
40 DEG.	0.0030216	0.0030450	0.0031151	0.0032322	0.0033952	0.0035988	0.0038278	0.0040513	0.0042210		
60 DEG.	0.0030216	0.0030441	0.0031116	0.0032242	0.0033810	0.0035768	0.0037971	0.0040122	0.0041758		
80 DEG.	0.0030216	0.0030435	0.0031092	0.0032193	0.0033728	0.0035648	0.0037814	0.0039935	0.0041548		
100 DEG.	0.0030216	0.0030433	0.0031090	0.0032189	0.0033723	0.0035643	0.0037808	0.0039927	0.0041541		
120 DEG.	0.0030216	0.0030437	0.0031108	0.0032230	0.0033793	0.0035746	0.0037943	0.0040090	0.0041722		
140 DEG.	0.0030216	0.0030445	0.0031138	0.0032299	0.0033916	0.0035934	0.0038203	0.0040415	0.0042095		
160 DEG.	0.0030216	0.0030451	0.0031166	0.0032365	0.0034040	0.0036137	0.0038502	0.0040813	0.0042570		
180 DEG.	0.0030216	0.0030454	0.0031177	0.0032393	0.0034094	0.0036230	0.0038645	0.0041012	0.0042816		

THETA 1=10 DEG.		HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.	0 DEG.	30 DEG.0
0 DEG.	0.0030454	0.0030704	0.0031456	0.0032721	0.0034504	0.0036765	0.0039363	0.0041962	0.0043983		
20 DEG.	0.0030454	0.0030700	0.0031441	0.0032684	0.0034429	0.0036630	0.0039137	0.0041619	0.0043527		
40 DEG.	0.0030454	0.0030691	0.0031404	0.0032596	0.0034259	0.0036340	0.0038687	0.0040987	0.0042740		
60 DEG.	0.0030454	0.0030681	0.0031363	0.0032503	0.0034092	0.0036076	0.0038312	0.0040499	0.0042165		
80 DEG.	0.0030454	0.0030674	0.0031337	0.0032446	0.0033994	0.0035930	0.0038114	0.0040253	0.0041882		
100 DEG.	0.0030454	0.0030673	0.0031335	0.0032443	0.0033990	0.0035926	0.0038109	0.0040246	0.0041872		
120 DEG.	0.0030454	0.0030678	0.0031357	0.0032494	0.0034079	0.0036058	0.0038287	0.0040466	0.0042123		
140 DEG.	0.0030454	0.0030687	0.0031394	0.0032578	0.0034230	0.0036293	0.0038617	0.0040887	0.0042612		
160 DEG.	0.0030454	0.0030695	0.0031428	0.0032658	0.0034379	0.0036540	0.0038983	0.0041380	0.0043205		
180 DEG.	0.0030454	0.0030698	0.0031441	0.0032690	0.0034443	0.0036650	0.0039156	0.0041622	0.0043506		

EVALUATION OF SURFACE INTEGRAL APPEARING IN RECEIVER/TRANSMITTER GAIN FOR VARIOUS ANTENNA POSITIONS

THETA 1=20 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	0 DEG.
0.0031177	0.0031444	0.0032251	0.0033616	0.0035554	0.0038041	0.0040941	0.0043897	0.0046241	
20 DEG.	0.0031177	0.0031439	0.0032230	0.0033563	0.0035448	0.0037847	0.0040615	0.0043397	0.0045570
40 DEG.	0.0031177	0.0031427	0.0032177	0.0033436	0.0035201	0.0037421	0.0039947	0.0042443	0.0044365
60 DEG.	0.0031177	0.0031412	0.0032119	0.0033301	0.0034951	0.0037018	0.0039354	0.0041649	0.0043405
80 DEG.	0.0031177	0.0031403	0.0032081	0.0033218	0.0034803	0.0036788	0.0039027	0.0041221	0.0042893
100 DEG.	0.0031177	0.0031402	0.0032080	0.0033216	0.0034802	0.0036786	0.0039024	0.0041215	0.0042882
120 DEG.	0.0031177	0.0031411	0.0032116	0.0033296	0.0034944	0.0037007	0.0039336	0.0041618	0.0043357
140 DEG.	0.0031177	0.0031424	0.0032171	0.0033425	0.0035180	0.0037385	0.0039883	0.0042341	0.0044220
160 DEG.	0.0031177	0.0031436	0.0032221	0.0033545	0.0035408	0.0037766	0.0040462	0.0043137	0.0045196
180 DEG.	0.0031177	0.0031441	0.0032242	0.0033594	0.0035504	0.0037934	0.0040727	0.0043515	0.0045671

THETA 1=30 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	0 DEG.
0.0032393	0.0032690	0.0033595	0.0035141	0.0037364	0.0040272	0.0043753	0.0047420	0.0050436	
20 DEG.	0.0032393	0.0032682	0.0033562	0.0035059	0.0037197	0.0039965	0.0043228	0.0046595	0.0049298
40 DEG.	0.0032393	0.0032663	0.0033480	0.0034859	0.0036806	0.0039280	0.0042131	0.0044994	0.0047233
60 DEG.	0.0032393	0.0032641	0.0033390	0.0034646	0.0036405	0.0038616	0.0041128	0.0043613	0.0045525
80 DEG.	0.0032393	0.0032627	0.0033333	0.0034514	0.0036164	0.0038229	0.0040562	0.0042849	0.0044593
100 DEG.	0.0032393	0.0032627	0.0033333	0.0034515	0.0036165	0.0038231	0.0040562	0.0042845	0.0044582
120 DEG.	0.0032393	0.0032641	0.0033391	0.0034648	0.0036406	0.0038616	0.0041121	0.0043589	0.0045477
140 DEG.	0.0032393	0.0032663	0.0033481	0.0034853	0.0036798	0.0039260	0.0042083	0.0044897	0.0047075
160 DEG.	0.0032393	0.0032683	0.0033561	0.0035051	0.0037173	0.0039901	0.0043081	0.0046306	0.0048841
180 DEG.	0.0032393	0.0032690	0.0033594	0.0035140	0.0037330	0.0040179	0.0043531	0.0046967	0.0049692

EVALUATION OF SURFACE INTEGRAL APPEARING IN RECEIVER/TRANSMITTER GAIN FOR VARIOUS ANTENNA POSITIONS

THETA I=40 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.
0 DEG.	0.0034094	0.0034438	0.0035495	0.0037321	0.0040001	0.0043607	0.0048097	0.0053080	0.0057443
20 DEG.	0.0034094	0.0034426	0.0035443	0.0037192	0.0039733	0.0043102	0.0047205	0.0051618	0.0055322
40 DEG.	0.0034094	0.0034396	0.0035315	0.0036877	0.0039104	0.0041977	0.0045352	0.0048922	0.0051604
60 DEG.	0.0034094	0.0034362	0.0035173	0.0036559	0.0038458	0.0040884	0.0043663	0.0046436	0.0048590
80 DEG.	0.0034094	0.0034341	0.0035084	0.0036330	0.0038070	0.0040249	0.0042713	0.0045131	0.0046978
100 DEG.	0.0034094	0.0034342	0.0035086	0.0036334	0.0038075	0.0040256	0.0042719	0.0045132	0.0046970
120 DEG.	0.0034094	0.0034365	0.0035180	0.0036549	0.0038471	0.0040901	0.0043675	0.0046429	0.0048551
140 DEG.	0.0034094	0.0034400	0.0035323	0.0036689	0.0039117	0.0041935	0.0045336	0.0048748	0.0051441
160 DEG.	0.0034094	0.0034431	0.0035452	0.0037202	0.0039735	0.0043071	0.0047084	0.0051306	0.0054741
180 DEG.	0.0034094	0.0034443	0.0035504	0.0037330	0.0039995	0.0043545	0.0047383	0.0052530	0.0056378

THETA I=50 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.
0 DEG.	0.0036230	0.0036639	0.0037910	0.0040143	0.0043505	0.0048208	0.0054408	0.0061861	0.0069005
20 DEG.	0.0036230	0.0036620	0.0037831	0.0039942	0.0043078	0.0047374	0.0052850	0.0059104	0.0064717
40 DEG.	0.0036230	0.0036575	0.0037635	0.0039453	0.0042081	0.0045539	0.0049709	0.0054139	0.0057815
60 DEG.	0.0036230	0.0036524	0.0037419	0.0038931	0.0041068	0.0043790	0.0046738	0.0050117	0.0052618
80 DEG.	0.0036230	0.0036492	0.0037283	0.0038610	0.0040464	0.0042789	0.0045419	0.0048005	0.0049981
100 DEG.	0.0036230	0.0036494	0.0037287	0.0038617	0.0040475	0.0042803	0.0045434	0.0048014	0.0049981
120 DEG.	0.0036230	0.0036530	0.0037432	0.0038752	0.0041098	0.0043830	0.0046980	0.0050141	0.0052603
140 DEG.	0.0036230	0.0036584	0.0037654	0.0039483	0.0042125	0.0045593	0.0049755	0.0054127	0.0057684
160 DEG.	0.0036230	0.0036631	0.0037854	0.0039976	0.0043122	0.0047408	0.0052807	0.0058313	0.0063978
180 DEG.	0.0036230	0.0036650	0.0037934	0.0040179	0.0043545	0.0048216	0.0054081	0.0060183	0.0067334

EVALUATION OF SURFACE INTEGRAL APPEARING IN RECEIVER/TRANSMITTER GAIN FOR VARIOUS ANTENNA POSITIONS

THETA 1=60 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.
0 DEG.	0.0038645	0.0039138	0.0000000	0.0043466	0.0047782	0.0054127	0.0063141	0.0075186	0.0088231
20 DEG.	0.0038645	0.0039112	0.0040573	0.0043163	0.0047119	0.0052765	0.0060404	0.0069844	0.0079089
40 DEG.	0.0038645	0.0039046	0.0040285	0.0042434	0.0045593	0.0049852	0.0055157	0.0061033	0.0066131
60 DEG.	0.0038645	0.0038971	0.0039969	0.0041662	0.0044070	0.0047166	0.0050787	0.0054497	0.0057458
80 DEG.	0.0038645	0.0038924	0.0039771	0.0041191	0.0043177	0.0045669	0.0048491	0.0051269	0.0053396
100 DEG.	0.0038645	0.0038927	0.0039777	0.0041202	0.0043192	0.0045690	0.0048515	0.0051288	0.0053405
120 DEG.	0.0038645	0.0038980	0.0039988	0.0041693	0.0044117	0.0047232	0.0050866	0.0054563	0.0057481
140 DEG.	0.0038645	0.0039059	0.0040314	0.0042483	0.0045070	0.0049965	0.0055300	0.0061142	0.0066096
160 DEG.	0.0038645	0.0039128	0.0040606	0.0043224	0.0047215	0.0052900	0.0060523	0.0069694	0.0078232
180 DEG.	0.0038645	0.0039156	0.0040721	0.0043531	0.0047663	0.0054252	0.0063145	0.0074421	0.0085519

THETA 1=70 DEG.									
HORIZONTAL LISTING=THETA 2					VERTICAL LISTING=PHI 2				
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.
0 DEG.	0.0041012	0.0041602	0.0043471	0.0046087	0.0052389	0.0060953	0.0074222	0.0094158	0.0118525
20 DEG.	0.0041012	0.0041566	0.0043311	0.0046460	0.0051416	0.0059830	0.0069567	0.0084061	0.0099605
40 DEG.	0.0041012	0.0041476	0.0042916	0.0045439	0.0049218	0.0054443	0.0061207	0.0069046	0.0076182
60 DEG.	0.0041012	0.0041375	0.0042484	0.0044372	0.0047077	0.0050546	0.0054742	0.0059063	0.0062564
80 DEG.	0.0041012	0.0041311	0.0042213	0.0043726	0.0045843	0.0048503	0.0051518	0.0054490	0.0056767
100 DEG.	0.0041012	0.0041314	0.0042220	0.0043737	0.0045861	0.0048526	0.0051246	0.0054514	0.0056781
120 DEG.	0.0041012	0.0041384	0.0042304	0.0044407	0.0047131	0.0050665	0.0054843	0.0059159	0.0062612
140 DEG.	0.0041012	0.0041491	0.0042448	0.0045496	0.0049313	0.0054601	0.0061432	0.0069285	0.0076280
160 DEG.	0.0041012	0.0041584	0.0043352	0.0046534	0.0051545	0.0059048	0.0069867	0.0084158	0.0098826
180 DEG.	0.0041012	0.0041622	0.0043515	0.0046967	0.0052530	0.0061183	0.0074421	0.0093387	0.0114658

EVALUATION OF SURFACE INTEGRAL APPEARING IN RECEIVER/TRANSMITTER GAIN FOR VARIOUS ANTENNA POSITIONS

THETA 1=80 DEG.		HORIZONTAL LISTING=THETA 2						VERTICAL LISTING=PHI 2	
0 DEG.	10 DEG.	20 DEG.	30 DEG.	40 DEG.	50 DEG.	60 DEG.	70 DEG.	80 DEG.	90 DEG.
0 DEG.	0.0042816	0.0043490	0.0045636	0.0049628	0.0056263	0.0067147	0.0085379	0.0115483	0.0155431
20 DEG.	0.0042816	0.0043446	0.0045437	0.0049084	0.0054978	0.0064178	0.0078342	0.0098922	0.0122522
40 DEG.	0.0042816	0.0043334	0.0044945	0.0047795	0.0052127	0.0058266	0.0066460	0.0076344	0.0085698
60 DEG.	0.0042816	0.0043209	0.0044410	0.0046462	0.0049416	0.0053279	0.0057902	0.0062765	0.0066753
80 DEG.	0.0042816	0.0043129	0.0044074	0.0045660	0.0047880	0.0050670	0.0053836	0.0056959	0.0059355
100 DEG.	0.0042816	0.0043131	0.0044079	0.0045670	0.0047895	0.0050691	0.0053861	0.0056980	0.0059364
120 DEG.	0.0042816	0.0043216	0.0044426	0.0046490	0.0049462	0.0053349	0.0057993	0.0062851	0.0066785
140 DEG.	0.0042816	0.0043346	0.0044972	0.0047841	0.0052207	0.0058401	0.0066673	0.0076593	0.0085828
160 DEG.	0.0042816	0.0043460	0.0045470	0.0049144	0.0055084	0.0064364	0.0078621	0.0099084	0.0121983
180 DEG.	0.0042816	0.0043506	0.0045671	0.0049692	0.0056378	0.0067334	0.0085519	0.0114658	0.0151546

IV. DETERMINATION OF SCATTERING CROSS SECTION FROM THE TABLES

From the tables in Section III, the values of scattering cross section for various angles of incidence and scattering can be determined readily. Even in cases where the pencil beam antennas used have somewhat different normalized gain functions, $f(\psi)$, these tables will still be fairly accurate due to the extremely narrow concentration of beam power.

The scattering cross section can be determined by measuring transmitted and received power with the following formula:

$$\sigma_o(\theta_1, \theta_2, \theta_3) = \frac{(4\pi)^3 L^2}{\lambda^2} \frac{P_R}{P_T G_{MR} G_{MT} I(\theta_1, \theta_2, \phi_2)} .$$

However, the more common and convenient procedure is to compare the received power to that received from a standard target having a known scattering cross section, σ_s , where σ_s represents an actual cross sectional area and is not dimensionless. In this case, the relationship between power transmitted and received from the standard target is

$$P_{RS} = P_T \frac{G_{MR} G_{MT} \lambda^2}{(4\pi)^3 L^4} \sigma_s ,$$

where the target is located an equal distance, L , from both transmitter and receiver. Multiplying the original expression of $\sigma_o(\theta_1, \theta_2, \phi_2)$ by this equation, one arrives at the relationship

$$\sigma_o(\theta_1, \theta_2, \phi_2) = \left(\frac{\sigma_s}{L^2} \right) \left(\frac{P_R}{P_{RS}} \right) \frac{1}{I(\theta_1, \theta_2, \phi_2)} .$$

Thus, the scattering cross section may easily be determined using these tables in conjunction with the easily measurable parameters P_R , P_{RS} , and σ_s . The tables are accurate to within about 5% and can even be used with pencil beam antennas with a gain function which varies somewhat from the mathematical model used here, since, in most cases, a high degree of accuracy in the determination of σ_o is neither practical or attainable.